

Form Factors in Relativistic Quantum Mechanics Approaches and Space-Time Translation Invariance

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Abstract

Invariance of form factors under Lorentz boosts is a criterion often advocated to determine whether their estimate in a RQM framework is reliable. It is shown that verifying relations stemming from covariance properties under space-time translations could be a more important criterion. Form factors calculated in different approaches for a simple system are discussed with these various respects. An approximate method is shown to remove the main discrepancies related to a violation of the above relations.

1 Introduction

There are many ways to implement relativity in the description of a few-body system and its properties, such as form factors. Ultimately, their predictions should converge but in an approximate calculation, some approaches may be more efficient than other ones. Relativistic quantum mechanics (RQM) has the advantage over field theory that it is dealing with a fixed number of (effective) degrees of freedom. As a counterpart, relativistic covariance properties are not trivially fulfilled. Following Dirac's work[1], several approaches depending on the symmetry properties of the hyper surface on which physics is formulated have been proposed: they are the instant, front, and point forms. These ones have been used for calculating form factors of various systems, evidencing large discrepancies in the approximation of a one-body current (see Ref. [2] and references therein). This raises the question of determining criteria allowing one to discriminate between the various approaches. Invariance under Lorentz boosts, which can be easily checked, is obviously one of them but relativity also implies other transformations such as space-time translations. The invariance under these ones provides the well-known conservation of energy and momentum, which, evidently, holds globally for the system. However, in an incomplete RQM calculation, this property is not necessarily fulfilled at the interaction vertex of its constituents with an external probe.

In the present contribution, we examined form factors of a simple system with respect to the above transformations. After showing the results obtained in different approaches (Sec. 2), we proceed to their discussion in view of relations which stem from the transformation of currents under space-time translations (Sec. 3).

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2 Form Factors in Different Forms: Single-Particle Approximation

The system whose form factors are studied here is a theoretical one. It corresponds to the ground state of the Wick-Cutkoski model (scalar particles exchanging a massless scalar meson). Due to a hidden symmetry, the Bethe-Salpeter equation for this system can be easily solved and the charge and scalar form factor can be calculated exactly. In some sense, this provides our “experiment”. It can be compared to form factors calculated in the single-particle current approximation for different RQM approaches and using the same solution of a mass operator. The two types of contributions are shown in Fig. 1, l.h.s. and r.h.s. respectively.

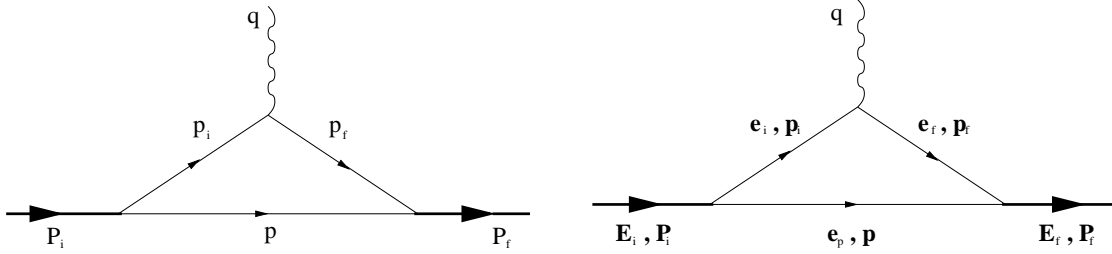


Figure 1: Single-particle current contribution in field-theory (l.h.s.) and RQM (r.h.s.) approaches. Intermediate particles are respectively off-mass shell and on-mass shell.

In the instant form, where results are not invariant under a Lorentz transformation (boost), form factors can be considered for the Breit frame (I.F. (Breit frame)) and for a frame where the initial- and final-state momenta are parallel while their sum goes to infinity (I.F. (parallel)). In the front form, where results are not invariant under a Lorentz transformation (rotation), they can be considered for the configuration $q^+ = 0$ (F.F. (perp.)) and for a configuration where the initial- and final-state momenta are parallel to the front orientation (F.F. (parallel)). Not surprisingly, the last results are identical to those in the instant form with an infinite average momentum. In the point-form, results turn out to be Lorentz invariant. Different versions may be nevertheless considered. The first one[3, 4], extensively used in many works, is an instant form displaying the same symmetry as the point form (“P.F.”). The second one[5] is more in the spirit of the Dirac point form, where physics is formulated on an hyperboloid surface (D.P.F.).

Results are presented in Fig. 2 for the charge form factor, $F_1(Q^2)$, at both low and high Q^2 . These ranges are aimed to point to the charge radius, $\langle r^2 \rangle$, and to the asymptotic behavior (expected to be Q^{-4}). In comparison to “experiment” (small diamonds in the figure), it is noticed that the instant-form (Breit-frame) and front-form ($q^+ = 0$) cases do rather well. Lorentz invariance, which underlies point-form results, does not imply good results. Apart from the above instant- and front-form cases, the charge radius scales like the inverse of the mass of the system, leading to the paradox that the former quantity goes to infinity when the latter goes to zero. This suggests the violation of some symmetry. Which one however?

3 Role of Space-Time Translation Invariance

Space-time translation invariance implies the 4-momentum conservation. This has to hold globally for any physical process, giving the relation $(P_i^\mu + q^\mu = P_f^\mu$ in the present case (Fig. 1)). In field-theory, such a relation also holds at the interaction vertex of the constituents with

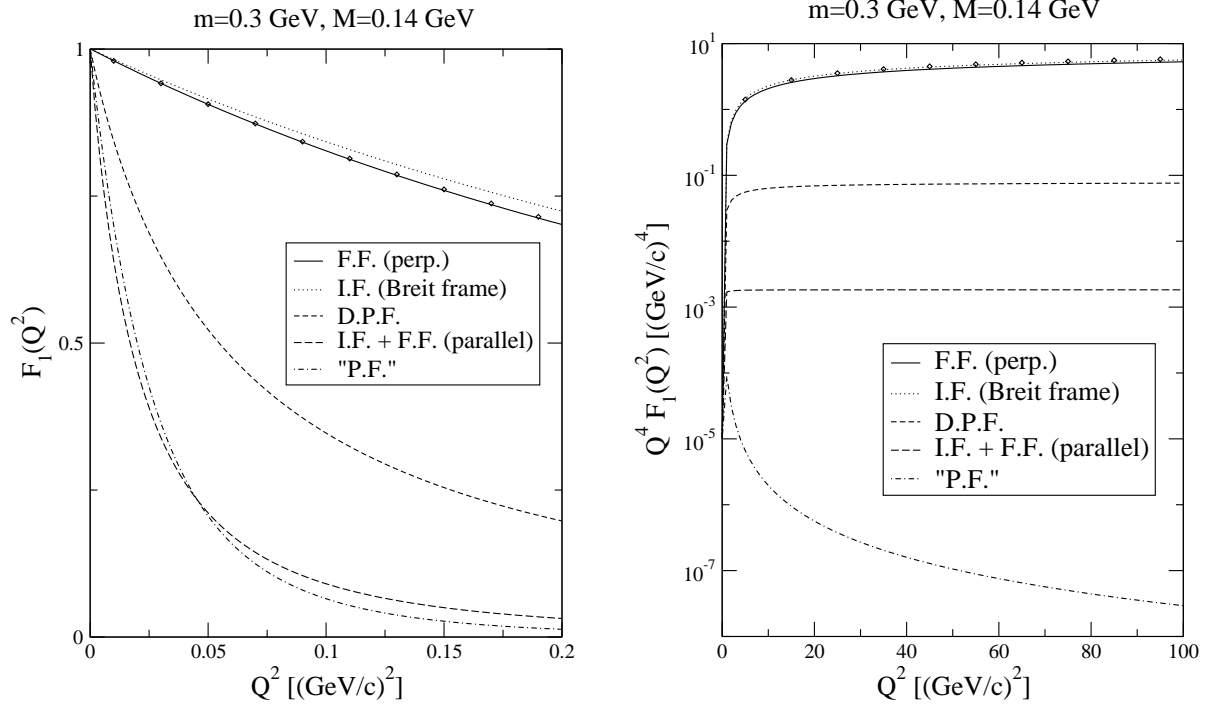


Figure 2: Form factors at low and large Q^2 (l.h.s. and r.h.s. respectively). Parameters are appropriate to a pion-like system. See text for the definition of abbreviations.

the external probe ($p_i^\mu + q^\mu = p_f^\mu$). However, the last one is not verified in RQM approaches, where constituents are on-mass shell. To quantitatively discuss the consequences of this feature, it is appropriate to consider relations that stem from the transformation of a current under space-time translations[6], which are generated by the operators of the Poincaré algebra, P^μ . Considering matrix elements, one should verify relations like:

$$\begin{aligned}
 \langle [P^\mu, J^\nu(x)] | \rangle &= -i \langle |\partial^\mu J^\nu(x)| \rangle, \\
 \langle [P_\mu, [P^\mu, J^\nu(x)]] | \rangle &= -\langle |\partial_\mu \partial^\mu J^\nu(x)| \rangle, \\
 \text{or, here, } \langle |q^2 J^\nu(x)| \rangle &= \langle |(p_i - p_f)^2 J^\nu(x)| \rangle,
 \end{aligned} \tag{1}$$

which are automatically fulfilled in field-theory but suppose the existence of many-body currents in RQM approaches. Assuming a single-particle current, it can be verified that the last equation is fulfilled exactly in the front-form ($q^+ = 0$) case or approximately in the instant-form (Breit frame) one. In all other cases, it is violated by large factors, from 30 up to 35000 ("P.F.") here.

Examination of these cases indicates that the factor multiplying Q^2 in calculations misses interaction effects. Correcting this factor such as to remove the above violation factors therefore provides an approximate way to account for the missing many-body currents required at all orders in the interaction to fulfill Eqs. 1. This approach has been applied to the form factors shown in Fig. 2. The resulting ones are presented in Fig. 3. It is observed that the largest discrepancies have vanished at low and high Q^2 . In particular, the peculiar behavior of form factors in the limit of a zero-mass system is completely removed.

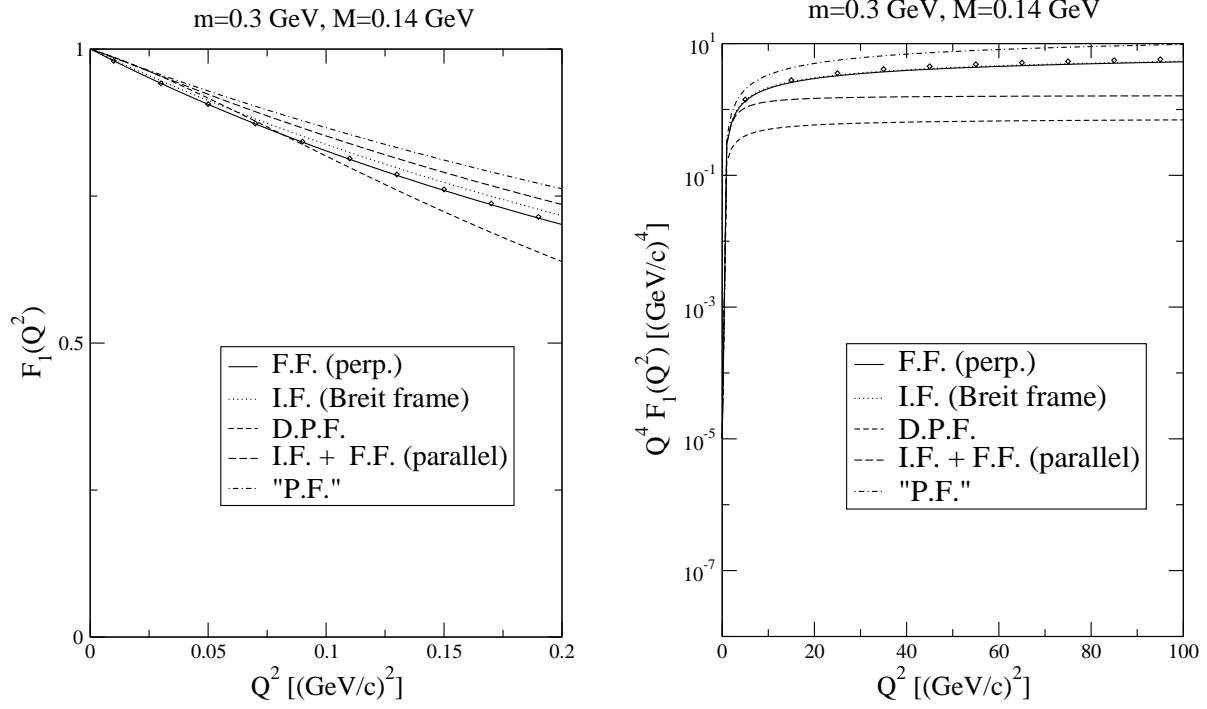


Figure 3: Form factors with corrections aimed to fulfill relations stemming from transformations of currents under space-time translations, at low and large Q^2 (l.h.s. and r.h.s. respectively).

4 Conclusion

The present work shows that constraints from space-time translation transformations could be more important than those from (homogeneous) Lorentz transformations. Invariance under these last ones turns out to be a disadvantage as there is no frame where the violation of the other constraints can be minimized. To a large extent, the present work confirms the standard front-form approach ($q^+ = 0$) and instant-form one (Breit-frame or $E_i = E_f$ case more generally) as more reliable frameworks in the single-particle current approximation.

References

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